P131

[1] $yz \perp x$-axis

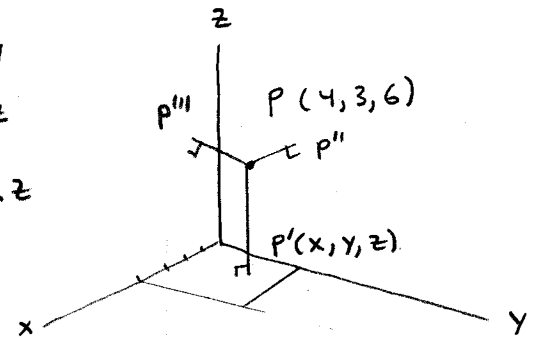
y-axis $\perp xz$

P132

[2] $P'(4,3,0)$ $\quad PP' \perp xy$

$P''(4,0,6)$ $\quad PP'' \perp yz$

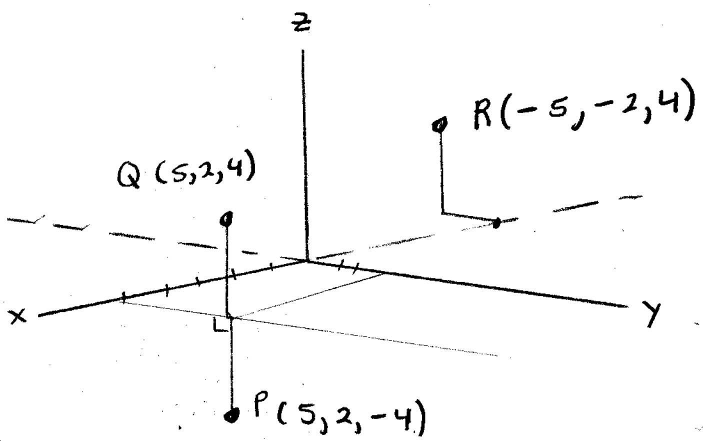
$P'''(0,3,6)$ $\quad PP''' \perp xz$



[3]

$Q(5,2,4)$

$R(-5,-2,4)$



[4.1] $(2-3,\ 3+1,\ 4-2) = (-1,4,2)$

[4.2] $(-2-3,\ -3+1,\ -4-2) = (-5,-2,-6)$

[4.3] $(-3-3,\ 1+1,\ -2-2) = (-6,2,-4)$

P133

[5] $4-\alpha = -1 \Rightarrow \alpha = 5$

$2-\beta = 3 \quad\quad \beta = -1$

$-3-\gamma = 5 \quad\quad \gamma = -8$

$\therefore (1,1,1) \longrightarrow (-4,2,9)$

[6] Intentionally left out

P134

[1.1] $d^2 = (3-2)^2 + (-2-0)^2 + (4-3)^2$

$= 1+4+1$

$= 6$

$\therefore\ d = \sqrt{6}$

[1.2] $d^2 = (0+4)^2 + (2-0)^2 + (3-1)^2$

$= 16+4+4$

$= 24$

$d = 2\sqrt{6}$

P135

[2.1] $(x-2)^2 + (y-0)^2 + (z+5)^2 = 36$

[2.2] To find the radius, note that the distance to yz-plane is 4. Then

$(x-4)^2 + (y+3)^2 + (z-5)^2 = 16$

P136

[3] $x^2 + y^2 + z^2 - 4x + 6y + 2z = 11$

$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 + 2z + 1) = 11 + 4 + 9 + 1$

$(x-2)^2 + (y+3)^2 + (z+1)^2 = 5^2$

$\therefore\ C(2,-3,-1),\ r = 5.$

[4] $x^2 + y^2 + z^2 = 16$

Interior $\{(x,y,z) \ni x^2 + y^2 + z^2 < 16\}$

Exterior $\{(x,y,z) \ni x^2 + y^2 + z^2 > 16\}$

[5] $C(2,1,-3),\ r = 1$

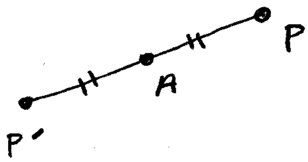
$C(2,1,-3),\ r = 4$

$R = \{(x,y,z) \ni 1 < (x-2)^2 + (y-1)^2 + (z+3)^2 < 16\}$

P 136 Exercises

[1] P' sym to $P(5,-2,6)$ w.r.t. $A(3,2,-4)$

A must be midpoint of $P'P$.



$\therefore$ P' (1 , 6 , -14)

[2] Get Pts on x- and y- axes that are equidistant from $A(4,5,3)$ and $B(3,-2,5)$.

P on x-axis $\equiv P(x,0,0)$

Q on y-axis $\equiv Q(0,y,0)$

x-coord

$$PA^2 = (x-4)^2 + (0-5)^2 + (0-3)^2 = x^2 - 8x + 16 + 29 + 9$$

$$PB^2 = (x-3)^2 + (0+2)^2 + (0-5)^2 = x^2 - 6x + 9 + 4 + 25$$

$$PA^2 = PB^2 \equiv x^2 - 8x + 54 = x^2 - 6x + 38$$

$$\equiv 2x - 16 = 0$$

$$\equiv \boxed{x = 8}$$

Y-COORD

$$QA^2 = (4-0)^2 + (5-y)^2 + (3-0) \equiv 16 + 25 - 10y + y^2 + 9$$

$$QB^2 = (3-0)^2 + (-2-y)^2 + (5-0)^2 \equiv 9 + 4 + 4y + y^2 + 25$$

$$QA^2 = QB^2 \equiv y^2 - 10y + 50 = y^2 + 4y + 38$$

$$\equiv 16y - 12 = 0$$

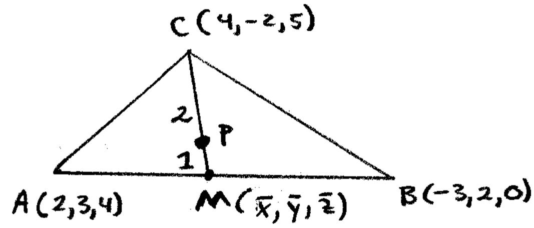
$$\equiv y = \frac{12}{16}$$

$$\equiv \boxed{y = \frac{3}{4}}$$

$\therefore P(8,0,0)$ and $Q\left(0, \frac{3}{4}, 0\right)$

P136, ctd

[3.1] Let $P(x,y,z)$ be the centroid of ΔABC.



Recall that the centroid is $\frac{2}{3}$ length of median from vertex.

Coords of M

$$\bar{x} = \frac{2-3}{2}, \quad \bar{y} = \frac{3+2}{2}, \quad \bar{z} = \frac{4+0}{2}$$

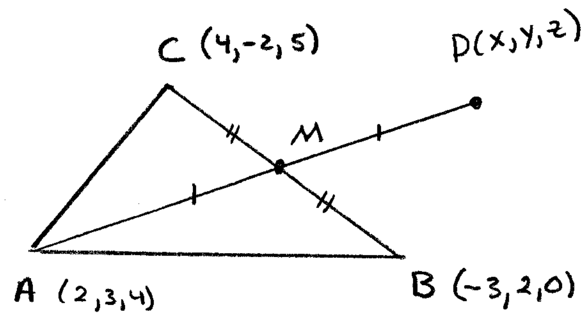
$$\boxed{M\left(-\tfrac{1}{2}, \tfrac{5}{2}, 2\right)}$$

Now $\frac{CP}{PM} = \frac{2}{3}$, so that

$$x = \frac{2(-1/2)+4}{3}, \quad y = \frac{2(\frac{5}{2})-2}{3}, \quad z = \frac{2(2)+5}{3}$$

$\therefore$ $P(1, 1, 3)$

[3.2]



M: $x = \frac{4-3}{2}, \quad y = \frac{-2+2}{2}, \quad z = \frac{5+0}{2}$

$$\boxed{M\left(\tfrac{1}{2}, 0, \tfrac{5}{2}\right)}$$

Let M be midPT AD,

$$\frac{1}{2} = \frac{x+2}{2}, \quad 0 = \frac{y+3}{2}, \quad \frac{5}{2} = \frac{4+z}{2}$$

$$x = -1, \quad y = -3, \quad z = 1$$

So

$\therefore$ $D(-1, -3, 1)$

pps 136-

p 137

[6.1]

center at (a, b, c)

$$a = \frac{-2+4}{2} = 1$$

$$b = \frac{1-3}{2} = -1$$

$$c = \frac{5-1}{2} = 2$$

$\boxed{C\ (1, -1, 2)}$

$$r^2 = (1-4)^2 + (-1+3)^2 + (2+1)^2$$

$$= 9 + 4 + 9$$

$$\boxed{r^2 = 22}$$

$$\therefore\ (x-1)^2 + (y+1)^2 + (z-2)^2 = 22$$

[6.2] Since $P(-5,1,4)$ on sphere, coordinates of center C are $(-a,b,c)$ where a,b,c all positive.

{ this says that center of sphere must lie on same Side of yz-plane as the point $P(-5,1,4)$. }

$r = a = b = c$ = distance from Center to $P(-5,1,4)$.

$(-a+5)^2+(b-1)^2+(c-4)^2 = r^2$, distance formula

$(-r+5)^2+(r-1)^2+(r-4)^2 = r^2$, substitute r for a,b,c

$r^2+r^2+r^2-10r-2r-8r+42 = r^2$

$2r^2-20r+42 = 0$

$r^2-10r+21 = 0$

$(r-3)(r-7) = 0$

$r=3$, $r=7$,

So center is $C(-3,3,3)$ or $C(-7,7,7)$.

$\therefore$ $(x+3)^2+(y-3)^2+(z-3)^2 = 9$

and $(x+7)^2+(y-7)^2+(z-7)^2 = 49$

are both solutions.

[7.1] $A(-1,0,0)$

$B(2,0,0)$

$$\frac{AP}{PB} = \frac{1}{2}.$$

Let $P(x, y, z)$ be a point on the figure. Then,

$$AP^2 = (x+1)^2 + y^2 + z^2$$

$$PB^2 = (x-2)^2 + y^2 + z^2$$

$$AP^2 = \frac{1}{4} PB^2.$$

$$x^2 + 2x + 1 + y^2 + z^2 = \frac{1}{4}(x^2 - 4x + 4 + y^2 + z^2)$$

$$4x^2 + 8x + 4 + 4y^2 + 4z^2 = x^2 - 4x + 4 + y^2 + z^2$$

$$3x^2 + 12x + 3y^2 + 3z^2 = 0$$

$$x^2 + 4x + y^2 + z^2 = 0$$

$$(x^2 + 4x + 4) + y^2 + z^2 = 4$$

$$\therefore \quad (x+2)^2 + (y-0)^2 + (z-0)^2 = 4$$

I.e. Figure is a circle radius 2 center at $(-2,0,0)$.

P137, ctd

[7.2] $A(1,2,0)$, $B(-1,4,2)$

$AP^2 + BP^2 = 38$

<u>soln</u>

Let $P(x,y,z)$ be point on figure.

$$AP^2 = (x-1)^2 + (y-2)^2 + z^2$$

$$BP^2 = (x+1)^2 + (y-4)^2 + z^2$$

$$AP^2 + BP^2 = 38$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + x^2 + 2x + 1 + y^2 - 8y + 16 + z^2 = 38$$

$$2x^2 + 2y^2 - 12y + 2z^2 + 22 = 38$$

$$x^2 + y^2 - 6y + z^2 = 8$$

$$(x-0)^2 + (y-3)^2 + (z-0)^2 = 8 + 9$$

$$\therefore \quad x^2 + (y-3)^2 + z^2 = 17$$

I.e. circle radius $\sqrt{17}$ center $(0,3,0)$.

p 147, ctd

[2.1] $\langle -2, 2, 3 \rangle \cdot \langle 4, 5, 6 \rangle$

$= (-2)(4) + (2)(5) + (3)(6)$

$= -8 + 10 + 18$

$= 20$

[2.2] $\vec{a} = 4\vec{e}_1 + 3\vec{e}_2 - \vec{e}_3$

$\vec{b} = -2\vec{e}_1 + \vec{e}_2 + 3\vec{e}_3$

$= (-2)(4) + (3)(1) + (-1)(3)$

$= -8 + 3 - 3$

$= -8$

p 148

[3.1] $\vec{a} = \langle -1, 0, 1 \rangle, \ \vec{b} = \langle -1, 2, 2 \rangle$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{(-1)(-1) + (0)(2) + (1)(2)}{\sqrt{2}\ \sqrt{9}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\therefore \ \theta = 45^\circ$

[3.2] $\vec{a} = \langle -3, 2, 1 \rangle, \ \vec{b} = \langle 2, 1, 4 \rangle$

$$\cos\theta = \frac{-6 + 2 + 4}{\sqrt{14}\ \sqrt{21}} = 0$$

$\therefore \ \theta = 90^\circ$

[1.1] $\vec{AB} \cdot \vec{AF}$

$= |\vec{AB}||\vec{AF}| \cos \theta$

$= (a)(a\sqrt{2}) \cos 45$

$= a^2 \sqrt{2} \left(\frac{\sqrt{2}}{2}\right)$

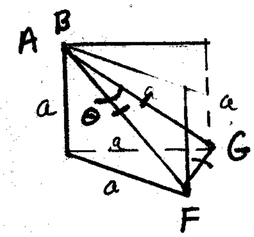
$= a^2$

[1.2] $\vec{AF} \cdot \vec{BG}$

$= |\vec{AF}||\vec{BG}| \cos 60^\circ$

$= (a\sqrt{2})(a\sqrt{2})\left(\frac{1}{2}\right)$

$= a^2$



$|\vec{AF}| = a\sqrt{2}$

$|\vec{BG}| = a\sqrt{2}$

$|\vec{FG}| = a\sqrt{2}$

$\therefore$ AFG EQUILATERAL Δ

Each of AF, BG, GF IS a diagonal of a square face Edge $\underline{a}$. So all equal.

[1.3]

$\vec{BG} \cdot \vec{DE}$

$= 0$, $\because$ BG $\perp$ DE

[1.4] $\vec{DE} \cdot \vec{FC}$

$= 1$, $\because$ $\vec{DE} \parallel \vec{FC}$

P 147

[4] $\vec{a} = \langle 2, -1, 4 \rangle$

$\vec{b} = \langle -4, 5, 3 \rangle$

§ $\vec{a} - k\vec{b} \perp \vec{a}$

Then

$$(\vec{a} - k\vec{b}) \cdot \vec{a} = 0$$

$$\vec{a} \cdot \vec{a} - k(\vec{a} \cdot \vec{b}) = 0$$

$$k = \frac{\vec{a} \cdot \vec{a}}{\vec{a} \cdot \vec{b}}$$

So that

$$k = \frac{\langle 2, -1, 4 \rangle \cdot \langle 2, -1, 4 \rangle}{\langle 2, -1, 4 \rangle \cdot \langle -4, 5, 3 \rangle}$$

$$= \frac{4 + 1 + 16}{-8 - 5 + 12}$$

$$= \frac{21}{-1}$$

$$= -21$$

$\therefore\ k = -21$

<u>CHECK</u> $\langle 2, -1, 4 \rangle + 21 \langle -4, 5, 3 \rangle = \langle -82, 104, 67 \rangle$

$$\langle -82, 104, 67 \rangle \cdot \langle 2, -1, 4 \rangle$$

$$= -164 - 104 + 268$$

$$= 0$$

P148 Exercises

[1.1] $2\vec{a} + \vec{x} = 3\vec{b}$

$$\vec{x} = 3\vec{b} - 2\vec{a}$$

$$= 3\langle 7, 9, -8\rangle - 2\langle 4, -2, 5\rangle$$

$$= \langle 13, 31, -34\rangle$$

$$\therefore \ x = \langle 13, 31, -34\rangle$$

[1, 2] $4\vec{x} - \vec{a} = 3\vec{a} - 4\vec{b} + 2\vec{x}$

$$2\vec{x} = 4\vec{a} - 4\vec{b}$$

$$x = 2(\vec{a} - \vec{b})$$

$$= 2\begin{bmatrix} 4-7 \\ -2-9 \\ 5+8 \end{bmatrix}$$

$$= 2\begin{bmatrix} -3 \\ -11 \\ 13 \end{bmatrix}$$

$$\therefore \ x = \langle -6, -22, 26\rangle$$

P 148 ctd

[2] $\vec{a} = \langle 1,1,0 \rangle$, $\vec{b} = \langle 1,0,1 \rangle$, $c = \langle 0,1,1 \rangle$, $\vec{p} = \langle 5,6,7 \rangle$

$\vec{p} = l\vec{a} + m\vec{b} + n\vec{c}$

$$l\begin{bmatrix}1\\1\\0\end{bmatrix} + m\begin{bmatrix}1\\0\\1\end{bmatrix} + n\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}5\\6\\7\end{bmatrix}$$

$$\begin{bmatrix} l + m \quad = 5 \\ l \quad + n = 6 \\ m + n = 7 \end{bmatrix} \equiv \begin{bmatrix} 1 & 1 & 0 & 5 \\ 1 & 0 & 1 & 6 \\ 0 & 1 & 1 & 7 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$\therefore \quad 2\vec{a} + 3\vec{b} + 4\vec{c} = \vec{p}$

P 149

[4] $\vec{a} = \langle 2,-1,-5 \rangle$, $\vec{b} = \langle 3x, 6, 4y-2 \rangle$, $c = \langle z-1, 2, z+1 \rangle$

[4.1] $\vec{a} \parallel \vec{b}$ iff $\vec{a} = t\vec{b}$, $t \in \mathbb{R}$.

$$t\begin{bmatrix}3x\\6\\4y-2\end{bmatrix} = \begin{bmatrix}2\\-1\\-5\end{bmatrix} \Rightarrow t = -\frac{1}{6}$$

then

$$-\frac{1}{6}\begin{bmatrix}3x\\6\\4y-2\end{bmatrix} = \begin{bmatrix}2\\-1\\-5\end{bmatrix} \Rightarrow \begin{bmatrix}3x\\6\\4y-2\end{bmatrix} = \begin{bmatrix}-12\\6\\30\end{bmatrix} \Rightarrow \begin{matrix} x = -4 \\ y = 8 \end{matrix}$$

$\therefore \quad x = -4, \ y = 8$

P149 ctd

[4,2] $\vec{a} \perp \vec{c}$ iff $\vec{a} \cdot \vec{c} = 0$

$$\begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} z-1 \\ 2 \\ z+1 \end{bmatrix} = 2(z-1) + (-1)(2) + (-5)(z+1)$$

$$= 2z - 2 - 2 - 5z - 5$$

$\vec{a} \perp \vec{c}$ iff $-3z - 9 = 0$

$$z = -3$$

$\therefore$ $z = -3$

[5] $\vec{a} = \langle x, 4, 6 \rangle$, $\vec{b} = \langle 2, y, 6 \rangle$, $\vec{c} = \langle 2, 4, z \rangle$. All $\perp$.

$\vec{a} \cdot \vec{b} = 0 \Rightarrow 2x + 4y + 36 = 0 \Leftrightarrow x + 2y + 18 = 0$

$\vec{a} \cdot \vec{c} = 0 \Rightarrow 2x + 16 + 6z = 0 \Leftrightarrow x + 3z + 8 = 0$

$\vec{b} \cdot \vec{c} = 0 \Rightarrow 4 + 4y + 6z = 0 \Leftrightarrow 2y + 3z + 2 = 0$

Then
$$\begin{bmatrix} 1 & 2 & 0 & -18 \\ 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -2 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4/3 \end{bmatrix}$$

$\therefore$ $\vec{a} = \langle -12, 4, 6 \rangle$, $\vec{b} = \langle 2, -3, 6 \rangle$, $\vec{c} = \langle 2, 4, \frac{4}{3} \rangle$

P149 ctd

[6] $\vec{a} = \langle 1, 2, -3 \rangle$, $\vec{b} = \langle 2, -1, -2 \rangle$, $\vec{x} = \langle x_1, x_2, x_3 \rangle$

where $\vec{x} \perp \vec{a}$ and $\vec{x} \perp \vec{b}$. Get x_1, x_2, x_3.

$$\vec{x} \perp \vec{a} \equiv \vec{x} \cdot \vec{a} = 0 \equiv x_1 + 2x_2 - 3x_3 = 0$$

$$\vec{x} \perp \vec{b} \equiv \vec{x} \cdot \vec{b} = 0 \equiv 2x_1 - x_2 - 2x_3 = 0$$

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -7/5 & 0 \\ 0 & 1 & -4/5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $x_3 = t$, $t \in \mathbb{R}$

$x_2 = \frac{4}{5} t$

$x_1 = \frac{7}{5} t$

P 149 ctd

[7] $\vec{a} = \langle 2, -2, 1 \rangle$ and $\vec{b} = \langle 2, 3, -4 \rangle$

[7.1] $\vec{c} = \vec{b} - k\vec{a}$, $k \in \mathbb{R}$. $\ni$ $\vec{a} \perp \vec{c}$. get k and $\vec{c}$.

$$\vec{a} \perp \vec{c} \iff \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} - k \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right) = 0$$

$$\iff \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 - 2k \\ 3 + 2k \\ -4 - k \end{bmatrix} = 0$$

$$\iff 2(2-2k) - 2(3+2k) + (-4-k) = 0$$

$$\iff 4 - 4k - 6 - 4k - 4 - k = 0$$

$$\iff -9k = 6$$

$$\iff k = -\frac{2}{3}$$

$$\therefore k = -\frac{2}{3}$$

[7.2] Get $\vec{u} \ni |\vec{u}| = 3$ and $\vec{u} \perp \vec{a}$ and $\vec{u} \perp \vec{b}$. Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$

$\vec{a} \perp \vec{u} \Rightarrow \vec{a} \cdot \vec{u} = 2u_1 - 2u_2 + u_3 = 0$

$\vec{b} \perp \vec{u} \Rightarrow \vec{b} \cdot \vec{u} = 2u_1 + 3u_2 - 4u_3 = 0$

$$\begin{bmatrix} 2 & -2 & 1 & 0 \\ 2 & 3 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{aligned} u_3 &= t \\ u_2 &= u_3 = t \\ u_1 &= \tfrac{1}{2} t \end{aligned}$$

since $|\vec{u}| = 3$, we have

$$u_1^2 + u_2^2 + u_3^2 = 9$$

$$\frac{t^2}{4} + t^2 + t^2 = 9$$

$$t^2 + 4t^2 + 4t^2 = 36$$

$$9t^2 = 36$$

$$t = 2$$

$$\therefore \vec{u} = \langle 1, 2, 2 \rangle$$

P 158

[1] $P_0(2,4,5)$, $\vec{n} = \langle 3,-1,-2 \rangle$

α: $3(x-2) - (y-4) - 2(z-5) = 0$

[2] $A(3,2,5)$, $B(4,-2,1)$. α through A, perp to $\vec{AB}$.

$\vec{AB} = \langle 1,-4,-4 \rangle$.

α: $(x-3) - 4(y-2) - (z-5) = 0$

[3] α through $(-5,-2,3)$ // xy-plane

β through $(-5,-2,3)$ // yz-plane

α : $(x+5) + (y+2) + z = 0$, $z \in \mathbb{R}$

β : $x + (y+2) + (z-3) = 0$, $x \in \mathbb{R}$

P159 ctd

[4] Plane through origin perpendicular to $\langle a, b, c \rangle$.

[5] $A(0,0,0)$, $B(1,0,-1)$, $C(0,2,3)$

$$\begin{array}{l} A: \\ B: \\ C: \end{array} \begin{bmatrix} d = 0 \\ a - c + d = 0 \\ 2b + 3c = 0 \end{bmatrix} \Longrightarrow \begin{array}{l} c = t \quad , \quad t \in \mathbb{R} \\ a = t \\ b = \frac{-3t}{2} \end{array}$$

Then,

$$tx - \frac{3t}{2} y + tz = 0$$

$$\therefore \boxed{2x - 3y + 2z = 0}$$

check

A: $2 \cdot 0 - 3 \cdot 0 + 2 \cdot 0 = 0$ ✓

B: $2(1) - 3(0) + 2(-1) = 0$ ✓

C: $2(0) - 3(2) + 2(3) = 0$ ✓

P 160

[6] $(2, 0, -4)$ and $P \parallel 3x + 4y - z = 5$

$3x + 4y - z - 5 = 0$

$$(3x - 5) + 4(y - 0) - (z - 0) = 0$$

$$3\left(x - \frac{5}{3}\right) + 4(y - 0) - (z - 0) = 0$$

$$\vec{n} = \langle 3, 4, -1 \rangle$$

$$\therefore 3(x - 2) - 4(y - 0) - (z + 4) = 0$$

$$3x - 4y - z - 6 - 4 = 0$$

$$\therefore \boxed{3x - 4y - z - 10 = 0}$$

P160

[7.1] $P(3,1,4)$, $\alpha: x-2y+2z+3=0$

$$d = \frac{|3-2+8+3|}{\sqrt{1+4+4}}$$

$$= \frac{12}{3}$$

$$\therefore d = 4$$

[7.2] $O(0,0,0)$, $\alpha: 3x+2y-4z+5=0$

$$d = \frac{|0+0+0+5|}{\sqrt{9+4+16}}$$

$$\therefore d = \frac{5}{\sqrt{29}}$$

[8] $\alpha: 2x-y+z=3 \Rightarrow \vec{n}_\alpha = \langle 2,-1,1\rangle$

$\beta: x+2y-4z=4 \Rightarrow \vec{n}_\beta = \langle 1,2,-4\rangle$

Let $\vec{u}=\langle u_1,u_2,u_3\rangle$ be direction vector of ℓ. Since $\ell \perp \vec{n}_\alpha$ and $\ell \perp \vec{n}_\beta$,

$$\langle 2,-1,1\rangle \cdot \langle u_1,u_2,u_3\rangle = 2u_1 - u_2 + u_3 = 0$$

$$\langle 1,2,-4\rangle \cdot \langle u_1,u_2,u_3\rangle = u_1 + 2u_2 - 4u_3 = 0$$

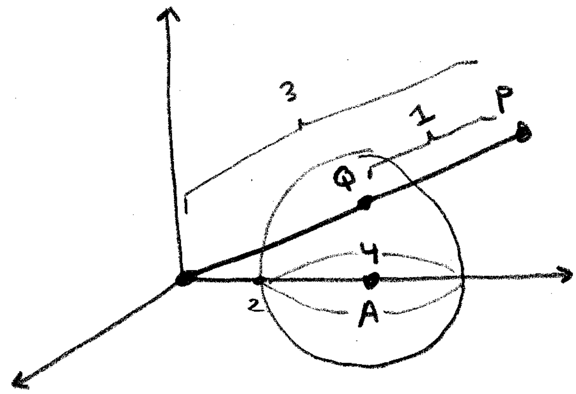
$$\begin{bmatrix} 2 & -1 & 1 & 0 \\ 1 & 2 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2/5 & 0 \\ 0 & 1 & -9/5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{aligned} u_3 &= t \\ u_2 &= \tfrac{9}{5}t \\ u_1 &= \tfrac{2}{5}t, \end{aligned}$$

so $\vec{u} = 5t\langle 2,9,5\rangle$ and the direction of ℓ is $\langle 2,9,5\rangle$ $\square$

which we knew from symmetric eqns of ℓ in demo 3.

P 163

[1]



P divides OQ externally in ratio 3 : 1

Let $\vec{p}, \vec{q}, \vec{a}$ be position vectors of P, Q, A.

(1) $|\vec{q} - \vec{a}| = 2$

$\vec{p} = \dfrac{3\vec{q} - 1\vec{o}}{3-1}$

(2) $\vec{p} = \frac{3}{2}\vec{q} \equiv \vec{q} = \frac{2}{3}\vec{p}$

(1,2) $\Rightarrow$ (3) $\left| \frac{2}{3}\vec{p} - \vec{a} \right| = 2$

$\frac{2}{3}\left| \vec{p} - \frac{3}{2}\vec{a} \right| = 2$

$\left| \vec{p} - \frac{3}{2}\vec{a} \right| = 3$,

But this is equation of circle center $\vec{c} = \frac{3}{2}\langle 0, 4, 0 \rangle = \langle 0, 6, 0 \rangle$ and radius 3

$\therefore$ Figure traced by P is Circle center $\langle 0, 6, 0 \rangle$ radius 3.

p 163, ctd

[2] $\vec{a} = \langle 4, -6, 8 \rangle$. sphere $|2\vec{p} - 3\vec{a}| = 6$.

$$|2\vec{p} - 3\vec{a}| = 6$$

$$2|\vec{p} - \tfrac{3}{2}\vec{a}| = 6$$

$$|\vec{p} - \tfrac{3}{2}\vec{a}| = 3$$

$$\tfrac{3}{2}\langle 4, -6, 8 \rangle = \langle 6, -9, 12 \rangle$$

$\therefore$ Center at $\langle 6, -9, 12 \rangle$, radius $= 3$

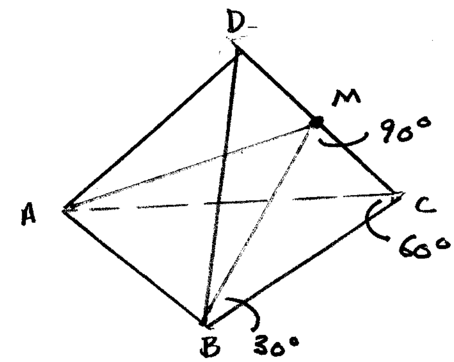
P 164

[1.1]

Since Δ ACD is one face of tetrahedro, $\angle A = \angle C = \angle D = 60°$.
M midpt of CD creates ΔAMC, a 30°-60°-90° triangle, with $\angle M = 90°$.
So $\vec{AM} \cdot \vec{CD} = 0$.

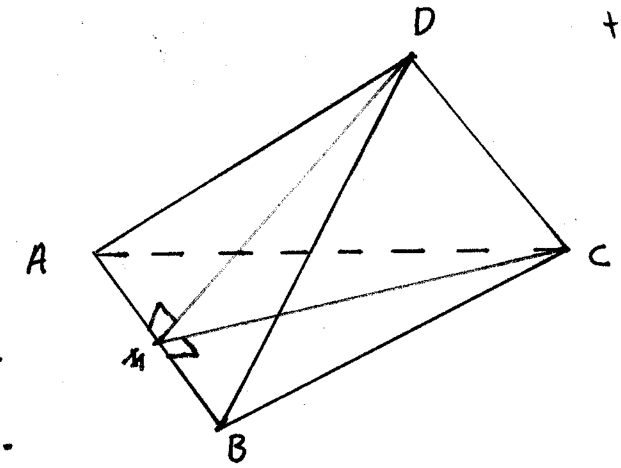
Similar reasonins leads to

$$\vec{BM} \cdot \vec{CD} = 0$$



[1.2] SOLUTION 1

ABCD regular tetrahedron



Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be Position vectors of A, B, C, D.

Let M be midpoint of AB.

Then $MC \perp AB$ and $MD \perp AB$, because $\triangle MBC$ and $\triangle MBD$ both $30°-60°-90°$. So $\vec{MC} \cdot \vec{AB} = 0$ and $\vec{MD} \cdot \vec{AB} = 0$.

This means

$$(\vec{c} - \vec{m}) \cdot \vec{AB} = 0 \qquad \text{(EQ1)}$$

$$(\vec{d} - \vec{m}) \cdot \vec{AB} = 0 \qquad \text{(EQ2)}$$

Subtracting EQ2 from EQ1

$$\left[(\vec{c} - \vec{m}) \cdot \vec{AB}\right] - \left[(\vec{d} - \vec{m}) \cdot \vec{AB}\right] = 0$$

$$\left[(\vec{c} - \vec{m}) - (\vec{d} - \vec{m})\right] \cdot \vec{AB} = 0$$

$$\left[\vec{c} - \vec{m} - \vec{d} + \vec{m}\right] \cdot \vec{AB} = 0$$

$$(\vec{c} - \vec{d}) \cdot \vec{AB} = 0$$

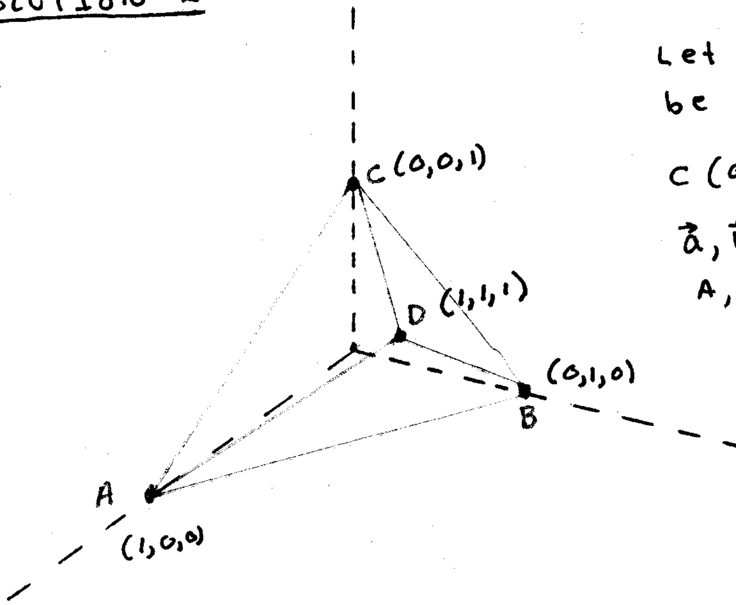
$$\vec{CD} \cdot \vec{AB} = 0$$

Therefor $CD \perp AB$.

$\square$

P164

[1.2] <u>SOLUTION 2</u>



Let vertices of tetrahedron be $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$, $D(1,1,1)$ as shown.

$\vec{a}, \vec{b}, \vec{c}, \vec{d}$ Position vectors of A, B, C, D.

$$\vec{AB} = \vec{b} - \vec{a} = \langle 0,1,0\rangle - \langle 1,0,0\rangle = \langle -1,1,0\rangle$$

$$\vec{CD} = \vec{d} - \vec{c} = \langle 1,1,1\rangle - \langle 0,0,1\rangle = \langle 1,1,0\rangle$$

$$\vec{AB} \cdot \vec{CD} = \langle -1,1,0\rangle \cdot \langle 1,1,0\rangle$$

$$= -1 + 1 + 0$$

$$= 0$$

$\therefore \quad AB \perp CD$

$\square$

P 164, ctd

[2.1] $P_0\,(2, -3, 7)$. $\vec{u} = \langle 1, 1, -4 \rangle$.

$$x - 2 = y + 3 = \frac{7 - z}{4}$$

[2.2] $2x - 6 = 4 - y = z - 5$

$$2(x - 3) = 4 - y = z - 5$$

$$x - 3 = \frac{4 - y}{2} = \frac{z - 5}{2}$$

$$x - 3 = \frac{y - 4}{-2} = \frac{z - 5}{2}$$

$\therefore$ direction vector is $\langle 1, -2, 2 \rangle$

[2.3] θ = angle of line of [2.1] and line of [2.2]

$$\cos\theta = \frac{\langle 1, 1, -4\rangle \bullet \langle 1, -2, 2\rangle}{|\langle 1, 1, -4\rangle|\,|\langle 1, -2, 2\rangle|}$$

$$= \frac{-9}{3\sqrt{2} \cdot 3}$$

$$= -\frac{1}{\sqrt{2}}$$

$$= \frac{-\sqrt{2}}{2}$$

$\therefore\ \theta = 135^\circ$

P 164, ctd

[3.1] $\quad \ell \quad \frac{x-1}{2} = \frac{2-y}{3} = z+2$

$\alpha \quad 3x - y - 2z = 12$

$$\ell \begin{cases} \frac{x-1}{2} = t \Rightarrow x = 2t+1 \\ \frac{2-y}{3} = t \Rightarrow y = 2-3t \\ z+2 = t \Rightarrow z = t-2 \end{cases}$$

{parametric EQNS of ℓ}

$3(2t+1) - (2-3t) - 2(t-2) = 12$

$6t + 3 - 2 + 3t - 2t + 4 = 12$

$7t + 5 = 12$

$\boxed{t = 1}$ {when $t = 1$, ℓ intersects α}

Then point of intersection is

$x = 2(1) + 1 = 3$

$y = 2 - 3(1) = -1$

$z = 1 - 2 = -1$

$\therefore$ $P(3, -1, -1)$ is point of intersection.

{It is easy to check this by substitution into EQNS for ℓ and α.}

p164, ctd

[3.2] Direction vector of ℓ is $\vec{u} = \langle 2, -3, 1 \rangle$

$\vec{n}$ of $\alpha = \langle 3, -1, -2 \rangle$

$$\cos\theta = \frac{\langle 2,-3,1\rangle \cdot \langle 3,-1,-2\rangle}{|\vec{u}||\vec{n}|}$$

$$= \frac{7}{\sqrt{14}\sqrt{14}}$$

$$= \frac{1}{2}$$

$\therefore \ \theta = 60^\circ$

[4.1] $x = 5$

[4.2] $-2(x-3) + 2(y+2) - 3(z-5) = 0$

$-2x + 6 + 2y + 4 - 3z + 15 = 0$

$\therefore \ -2x + 2y - 3z + 25 = 0$

[4.3] $3(x-4) + 6(y+2) - 4(z-3) = 0$

$3x + 6y - 4z - 12 + 12 + 12 = 0$

$\therefore \ 3x + 6y - 4z + 12 = 0$

[4.4] $\vec{a} = \langle 0,4,0\rangle - \langle 3,0,0\rangle = \langle -3,4,0\rangle$

$\vec{b} = \langle 0,0,5\rangle - \langle 3,0,0\rangle = \langle -3,0,5\rangle$

$$\vec{n} = \begin{vmatrix} i & j & k \\ -3 & 4 & 0 \\ -3 & 0 & 5 \end{vmatrix} = 20i + 15j + 12k$$

Plane: $20(x-0) + 15(y-4) + 12(z-0) = 0$

$\therefore \ 20x + 15y + 12z - 60 = 0$

[5] $\alpha: \quad 3x + z - 1 = 0$

$\beta: \quad x - \sqrt{5}\, y + 2z = 0$

$\vec{n}_\alpha = \langle 3, 0, 1 \rangle$

$\vec{n}_\beta = \langle 1, -\sqrt{5}, 2 \rangle$

$$\cos \theta = \frac{\langle 3,0,1\rangle \cdot \langle 1, -\sqrt{5}, 2\rangle}{|\langle 3,0,1\rangle|\,|\langle 1, -5, 2\rangle|}$$

$$= \frac{5}{\sqrt{10}\sqrt{10}}$$

$$= \frac{1}{2}$$

$\therefore \quad \theta = 60^\circ$

[6] <u>Solution 1</u> (uses cross product)

$P_0(-2,1,3)$

$\alpha: \; x - y + z = 0 \qquad \vec{n}_\alpha = \langle 1, -1, 1 \rangle$

$\beta: \; 2x + 3y - z = 5 \qquad \vec{n}_\beta = \langle 2, 3, -1 \rangle$

Desire plane $\gamma \perp \alpha$, $\gamma \perp \beta$, P_0 in γ, normal vector $\vec{n}_\gamma$

$$\vec{n}_\gamma = \vec{n}_\alpha \times \vec{n}_\beta$$

$$= \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \langle -2, 3, 5 \rangle$$

then

$$\gamma: \quad -2(x+2) + 3(y-1) + 5(z-3) = 0$$

$$-2x + 3y + 5z - 4 - 3 - 15 = 0$$

$$2x - 3y - 5z + 22 = 0$$

$$\therefore \quad \gamma: \quad 2x - 3y - 5z + 22 = 0$$

$\square$

[6] <u>Solution 2</u>

$P_0\ (-2, 1, 3)$

$\alpha \quad x - y + z = 0 \qquad \vec{n}_\alpha = \langle 1, -1, 1\rangle \qquad \vec{n}_\gamma = \langle a, b, c\rangle$

$\beta \quad 2x + 3y - z = 5 \qquad \vec{n}_\beta = \langle 2, 3, -1\rangle$

$\gamma: \quad a(x+2) + b(y-1) + c(z-3) = 0$

$\gamma \perp \alpha \Rightarrow \vec{n}_\alpha \cdot \vec{n}_\gamma = 0 \Rightarrow \langle 1,-1,1\rangle \cdot \langle a,b,c\rangle = 0 \Rightarrow a - b + c = 0$

$\gamma \perp \beta \Rightarrow \vec{n}_\beta \cdot \vec{n}_\gamma = 0 \Rightarrow \langle 2,3,-1\rangle \cdot \langle a,b,c\rangle = 0 \Rightarrow 2a + 3b - c = 0$

$$\begin{array}{r} a - b + c = 0 \\ 2a + 3b - c = 0 \\ \hline 3a + 2b = 0 \end{array}$$

Let $\quad a = t, \ t \in \mathbb{R}$

then $\quad b = -\frac{3}{2} t$

$\qquad c = -\frac{5}{2} t$

Since we are interested only in the direction of $\langle a, b, c\rangle$, choose a convienent value for t; e.g. $t = 2$

$t = 2 \Rightarrow \langle a, b, c\rangle = \langle 2, -3, -5\rangle$.

Then

$\gamma: \quad 2(x+2) - 3(y-1) - 5(z-3) = 0$

$\qquad 2x - 3y - 5z + 4 + 3 + 15 = 0$

$\therefore \ \gamma: \quad 2x - 3y - 5z + 22 = 0$

$\square$

[2] $C(3,7,4)$, $r = 5$

EQN CIRCLE $(x-3)^2 + (y-7)^2 + (z-4)^2 = 25$

At intersection of xy-plane, $z = 0$.

$$(x-3)^2 + (y-7)^2 + (-4)^2 = 25$$

$$x^2 - 6x + y^2 - 14y + 9 + 49 + 16 = 25$$

$$x^2 - 6x + y^2 - 14y = -49$$

$$(x^2 - 6x + 9) + (y^2 - 14y + 49) = -49 + 49 + 9$$

$$(x-3)^2 + (y-7)^2 = 9$$

$\therefore$ circle in xy-plane $C(3,7,0)$, $r = 3$.

At intersection of yz-plane, $x = 0$

$$(-3)^2 + (y-7)^2 + (z-4)^2 = 25$$

$$y^2 - 14y + z^2 - 8z + 9 + 49 + 16 = 25$$

$$y^2 - 14y + z^2 - 8z = -49$$

$$(y-7)^2 + (z-4)^2 = -49 + 49 + 16$$

$\therefore$ Circle in yz-plane $C(0,7,4)$, $r = 4$

[3.1] $A(2,3,-4)$, $B(3,1,-1)$, $C(m, 7, n-1)$

colinear iff $\vec{AB} = t\,\vec{BC}$

$$\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = t \begin{bmatrix} m-3 \\ 7-1 \\ n \end{bmatrix}$$

since $-2 = 6t$, $t = -\frac{1}{3}$.

Then $1 = -\frac{1}{3}(m-3)$

$-3 = m-3$

$\boxed{m = 0}$

$3 = tn$

$3 = -\frac{1}{3}n$

$\boxed{n = -9}$

$\therefore \quad m = 0,\ n = -9$

[3.2] SOLUTION 1

$P(4,-2,5)$, $Q(-3,4,-4)$, $R(1,2,4)$, $S(m,1-m,4)$.

P, Q, R in Plane, so

$$\begin{aligned} 4x-2y+5z+d&=0\\ -3x+4y-4z+d&=0\\ x+2y+4z+d&=0 \end{aligned} \equiv \begin{bmatrix} 4 & -2 & 5 & 1 & 0\\ -3 & 4 & -4 & 1 & 0\\ 1 & 2 & 4 & 1 & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 1 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & \frac{2}{3} & 0\\ 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix} \equiv \begin{bmatrix} a = -d\\ b = \frac{-2d}{3}\\ c = \frac{d}{3} \end{bmatrix}$$

Plane

$$-dx - \frac{2d}{3}y + \frac{d}{3}z + d = 0$$

$$-x - \frac{2}{3}y + \frac{1}{3}z + 1 = 0$$

$$x + \frac{2}{3}y - \frac{1}{3}z - 1 = 0$$

$$3x + 2y - z - 3 = 0$$

for $\langle m, 1-m, 4\rangle$

$$3m + 2(1-m) - 4 - 3 = 0$$

$$3m + 2 - 2m - 7 = 0$$

$$m - 5 = 0$$

$\therefore\ m = 5$

[3.2] Solution 2 (using cross product)

$\overrightarrow{PQ} = \langle -3, 4, -4\rangle - \langle 4, -2, 5\rangle = \langle -7, 6, -9\rangle$

$\overrightarrow{PR} = \langle 1, 2, 4\rangle - \langle 4, -2, 5\rangle = \langle -3, 4, -1\rangle$

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -7 & 6 & -9 \\ -3 & 4 & -1 \end{vmatrix} = \langle 30, 20, -10\rangle = \langle 3, 2, -1\rangle$$

Plane $3(x+3) + 2(y-4) - (z+4) = 0$

$3x + 2y - z + 9 - 8 - 4 = 0$

$3x + 2y - z - 3 = 0$

m in plane

$3m + 2(1-m) - 4 - 3 = 0$

$3m - 2m + 2 - 4 - 3 = 0$

$m - 5 = 0$

$\therefore \quad m = 5$

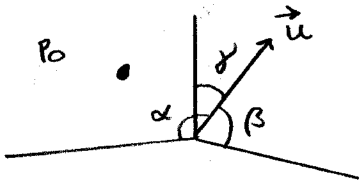
[4] $P_0(3,-1,2)$ on ℓ. Direction cosines are $\cos 60$, $\cos 45$, $\cos 60$.

$\ell \parallel \vec{u}$, $|\vec{u}| = 1$.

Then $u_1 = |\vec{u}| \cos 60 = \frac{1}{2}$

$u_2 = |\vec{u}| \cos 45 = \frac{\sqrt{2}}{2}$

$u_3 = |\vec{u}| \cos 60 = \frac{1}{2}$



ℓ: $\langle 3, -1, 2 \rangle + t \langle \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2} \rangle$

$$\begin{aligned} x &= 3 + \tfrac{1}{2} t \\ y &= -1 + \tfrac{\sqrt{2}}{2} t \\ z &= 2 + \tfrac{1}{2} t \end{aligned} \iff \begin{aligned} t &= 2(x-3) \\ t &= \frac{2(y+1)}{\sqrt{2}} = \sqrt{2}\,(y+1) \\ t &= 2(z-2) \end{aligned}$$

so ℓ: $2(x-3) = \sqrt{2}(y+1) = 2(z-2)$

[5] l_1 and l_2 each parallel to $<2, 3, 1>$.

To find eqn plane, use eqns for l_1, l_2 to find 3 non co-linear points in plane.

l_1 $\quad x = 2t, \; y = 3t + 2, \; z = t - 4$

$t = 0 \Rightarrow P_1(0, 2, -4)$

$t = 1 \Rightarrow P_2(2, 5, -3)$

l_2 $\quad x = 2t + 1, \; y = 3t, \; z = t$

$t = 0 \Rightarrow P_3(1, 0, 0)$.

Since P_1, P_2, P_3 in plane,

$$\begin{bmatrix} 2x + 5y - 3z + d = 0 \\ 2y - 4z + d = 0 \\ x + d = 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}$$

$$\equiv \quad d = t, \; c = \frac{t}{2}, \; b = \frac{t}{2}, \; a = -t$$

$$-tx + \frac{t}{2}y + \frac{t}{2}z - 1 = 0 \qquad \text{EQN Plane}$$

$\therefore \quad 2x - y - z - 2 = 0$ is EQN of Plane

[6] $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z+2}{5}$, $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z+2}{3}$

$x = 3t+1$ $\quad$ $x = 2t+1$

$y = 4t+3$ $\quad$ $y = 5t+3$

$z = 5t-2$ $\quad$ $z = 3t-2$

$3t+1 = 2t+1 \Rightarrow t = 0$

$4t+3 = 5t+3 \Rightarrow t = 0$

$5t-2 = 3t-2 \Rightarrow t = 0$

So ℓ_1 intersects ℓ_2 at $(x, y, z) = (1, 3, -2)$

$\ell_1 \parallel \langle 3,4,5 \rangle$, $\ell_2 \parallel \langle 2,5,3 \rangle$, $\vec{n} = \langle a,b,c \rangle$ = normal to plane

$\vec{n} = \langle 3,4,5 \rangle \times \langle 2,5,3 \rangle = \langle -13, 1, 7 \rangle$

SO EQN PLANE, USIN $P(1,3,-2)$, $\vec{n} = \langle -13, 1, 7 \rangle$ is,

$-13(x-1) + (y-3) + 7(z+2) = 0$

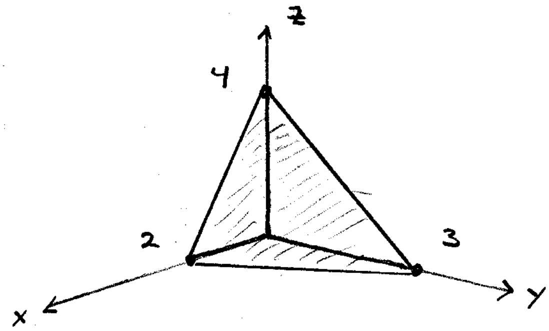
$-13x + y + 7z + 13 - 3 + 14 = 0$

$-13x + y + 7z + 24 = 0$

$\therefore \quad 13x - y - 7z - 24 = 0$

P 166

[7]



$V_{pyramid} = \frac{1}{3} B h$

Take base as the triangular region in xy-plane

$A_{\Delta} = \frac{1}{2}(2)(3) = 3$ sq units

$h = 4$ units

$\therefore \; V = \frac{1}{3}(3)(4) = 4 \text{ units}^3$

[8] $\alpha : \; 4x + 3y + 5z = 50$

[8,1] d $O(0,0,0)$ to α.

$$d = \frac{|0 + 0 + 0 - 50|}{\sqrt{16+9+25}}$$

$$= \frac{50}{5\sqrt{2}}$$

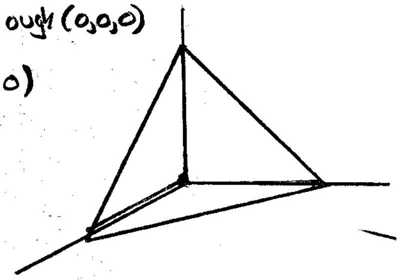
$$= \frac{10}{\sqrt{2}}$$

$$= \frac{10\sqrt{2}}{2}$$

$$\therefore \; \boxed{d = 5\sqrt{2}}$$

[8.2] Point $P(x, y, z)$ sym to $(0,0,0)$ w.r.t α.

P on line perpendicular to α through $(0,0,0)$
a distance of $2 \cdot 5\sqrt{2}$ from $(0,0,0)$



$$5\sqrt{2} = \frac{4x + 3y + 5z}{\sqrt{16+9+25}}$$

P on $\langle 0,0,0 \rangle + t \langle 4,3,5 \rangle$

$$x = 4t, \quad y = 3t, \quad z = 5t$$

Then,

$$\frac{4(4t) + 3(3t) + 5(5t)}{5\sqrt{2}} = 5\sqrt{2}\,(2)$$

$$16t + 9t + 25t = 25 \cdot 2 \cdot 2$$

$$50t = 100$$

$$t = 2$$

when $t = 2$

$$x = 8, \quad y = 6, \quad z = 10$$

$\therefore$ $P(8, 6, 10)$